Topology-constrained synthesis of vector patterns

Shizhe Zhou
USTC

Changyun Jiang
USTC

Sylvain Lefebvre*
INRIA
Motivation:

- Structured continuous vector pattern on curves.
- By-example: resembling and variations
- Physical objects that can be fabricated correctly on 3d printers (chairs, wristband, etc.)
demo
Input: SGPH (Simple General Polygon with Holes)
Synthesis Mechanism: Slice and Shuffle

Slice

Shuffle
Dynamic Programming
A Path from the first piece to the last piece gives a new vector pattern.
Merging cost

(a)

(b)
Merging cost

\[ D(i, j) = d_{match}(i, j) \cdot \alpha + (1 - \alpha) \cdot d_{tangent}(i, j) \]

\[ d_{tangent}(i, j) = \sum_{k=0}^{M-1} ||1 - \vec{g}_{r,i}^k \cdot \vec{g}_{l,j}^k|| \]

\[ d_{match}(i, j) = \min_{\tau} \left( \frac{1}{M} \sum_{k=0}^{M-1} ||y_{l,j}^k - y_{r,i}^k + \tau||^2 \right) \]
A Simple Example
Wait! There are “Islands” in the water..

“island”: regions enclosed in but disconnected from the outer border
and.. Separate parts 😞
Ignore topology → broken results
Challenges

• Previous synthesis problem.
  – variety, resemblance
  – interactive feedback, efficient displaying, zoom in/out

• New Challenge!
  – avoiding broken results
  – output data format: domain representation
Surprises

• Topology improves aesthetics.
  
  “Topology is part of the Structural quality”

• Users fully control topology by parameters:
  – given #connected-components and #holes
Our solution: Topology-constrained Synthesis

• Analysis from topology
  – Shape analysis for optimal piece-sampling

• Synthesis in two stages
  – Topology solver
  – Geometry solver
Topology descriptor:
Capturing topology changes

[Diagram showing the process of capturing topology changes with matrices and graphs]
Counting the inner holes

#hole ++
Tracking Topology
On-the-fly topology tracking

- Topology descriptor can not be pre-computed.
Topology equivalent: \( \equiv \)

Given two DP items \( a \ b \).

\[
\text{if}(a \ \text{and} \ b \ \text{have} \\
\begin{align*}
\text{the same } \#\text{Componets,} \\
\text{the same } \#\text{inner holes,} \\
\text{the same portal-component correspondences}
\end{align*}
\)

\[
\{ \\
\text{a is topoEqual to } b; \ /*\text{denoted as } a \equiv b*/ \\
\}
\]
Cubic DP

- Allowing arbitrary topology configurations co-exist

Topological descriptors

Index of pieces

Length of sequence

$T[i][j][k]$

Compare (Topology && Cost)
Cubic DP

if \( a \equiv T[i][j][k] \)
{
    if (a. cost is smaller)
        \( T[i][j][k] = a \);
    else
        discard a;
}
else
    \( T[i][j].add(a) \);
Backtracking

• The cubic DP gives complete solution space, where we explore by backtracking

• Simple UI: users specify #holes and #components

• O(K), K is the number of all possible solution.
Patterns on arbitrary curve

- Constituent pieces in parameterization domain
Closed loop

One more virtual piece to close

Bi-directional descriptor, but still $O(n\log(n))$ 😊

Topo-Descriptor can detect if the pattern is literally “closed”
Repetition

How: by a new cost energy? Jittering? Or a function to bring complex structures?

We expect variety
Variation control

Autocorrelation of index sequence:

\[ A(\eta_0...\eta_K) = \max_{j \in [0,K]} \left( \sum_{i \in [0,K-j]} e^{-\frac{|\eta_i - \eta_{i+j}|}{\sigma}} \right) \]
Variation control
Variation control
Geometry Solver

• Positioning pieces

• Gap stitching

• Global band fairing
Gap stitching

- linear rotation invariant deformation
  - Endpoint driven
  - Reorient and merge seams in one step

[Lipman, et al. 2005]
Our solution: Topology-constrained Synthesis

• **Analysis from topology**
  – Shape analysis for optimal piece-sampling

• **Synthesis in two stages**
  – Topology solver
  – Geometry solver
Shape analysis

• Uniform slicing
  – Simple but major drawback

• Non-uniform slicing
  – Best split line searching

*Slicing: important to the quality of the final results*
Uniform slicing

• Pros:
  - no need to accumulate band length → lighter DP
  - Variable “N” becomes explicitly known

• Cons
  - Trivial pieces → severe trivial repetition
  - Fixed group of topo-events → reduced topology variation
Problematic Piece

• Pieces contain no topology event
• Piece contain more than 1 topology events

(type A)

(type B)
Correct Piece

- Pieces contain exactly one topo-event
Non-uniform Slicing

- Reeb graph for optimal slicing
  - A discretization for analyzing topology of a continuous manifold
  - Tool for extracting topology information
  - Efficient
means “closed loop” constraint is imposed
Synthesizing rectangular patterns
Decorating surfaces

- Sketching patterns directly on a mesh
- Construct 3D bands along prescribed curves
Printed objects

Volumetric modeling Software: [IceSL, Sylvain Lefebvre 2013]
3D printing: Fused Filament Fabrication (FFF), Plastics Filament Fab (PLA), ZCorp powder based printer.
Inverse Patterns

#holes = 5

#holes = 0
Pure repetition

Strictly repeated
Summary

Input exemplars → single-connected vector shape → analysis for topology → Fabricated object
Limitation & Future work

• Stochastic synthesis of vector patterns
  – random exploration of topology descriptor

• Performance
  – accelerating dynamic programming
• Acknowledgement

We would like to thank colleagues in USTC and Inria for comments.

We would like to thank anonymous reviewers for their suggestions

• Funding

– National Science Foundation of China: No.61303147
– ERC grant ShapeForge (StG-2012-307877)
Thanks!

Welcome