序列和集合的算法II

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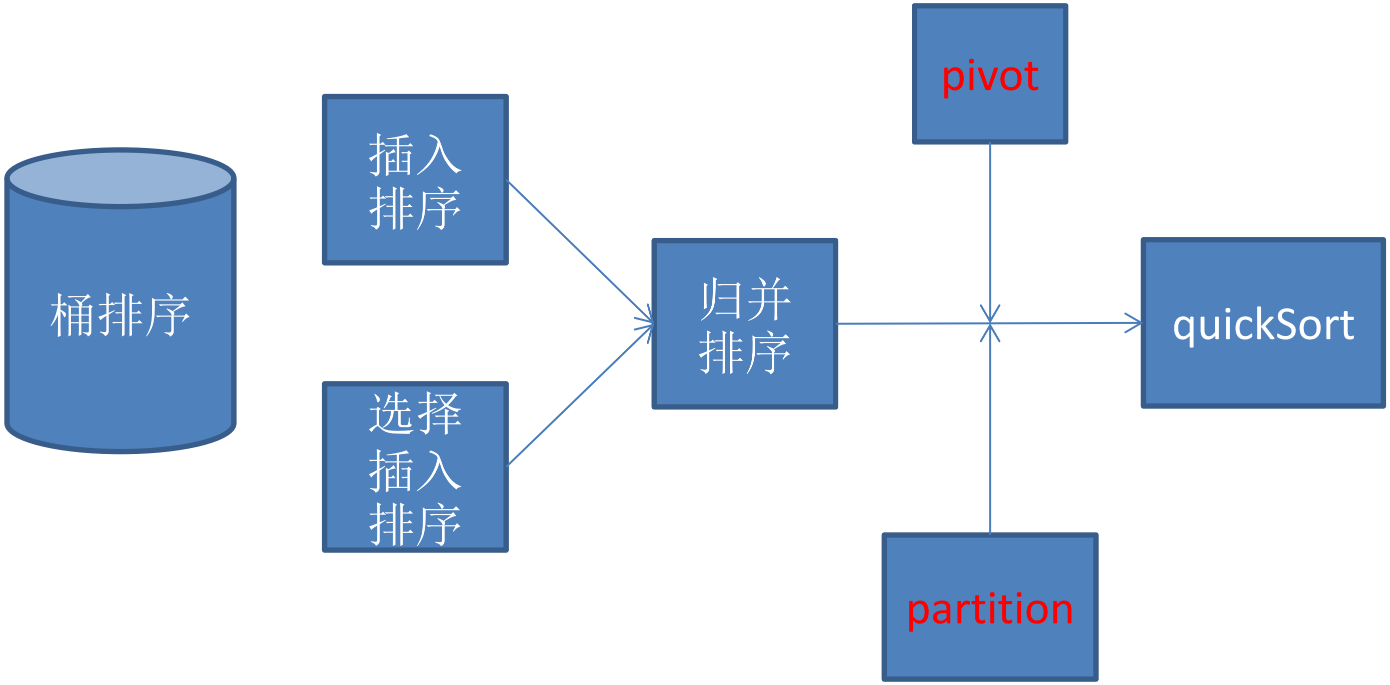
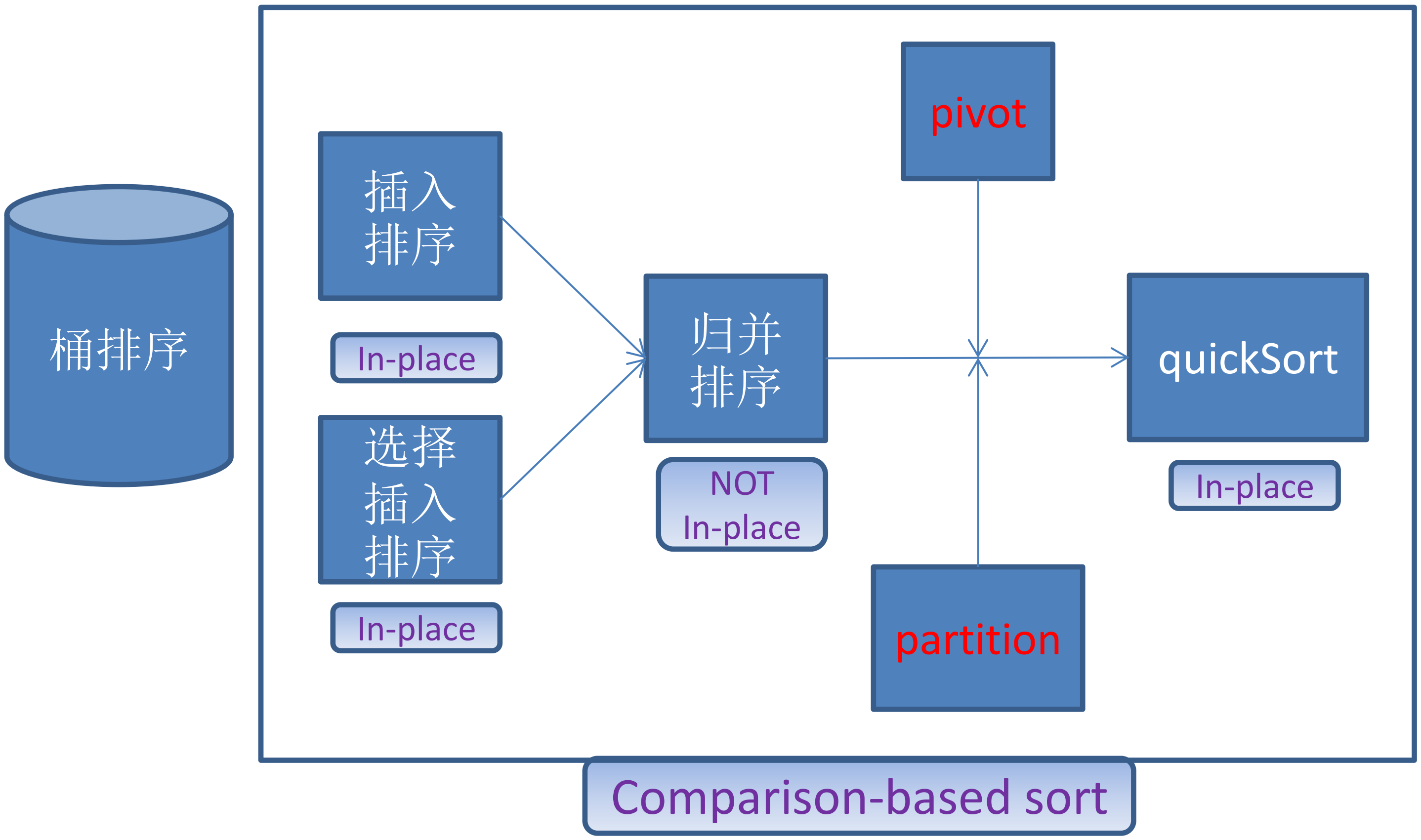
Course Code:00125401

排序算法



排序算法



桶排序
插入排序
In-place
选择插入排序
In-place
归并排序
NOT In-place
pivot
partition
quickSort
In-place
Comparison-based sort

桶排序和基数排序

Bucket and Radix Sort

算法 *Straight_Radix* (*X*, *n*, *k*)

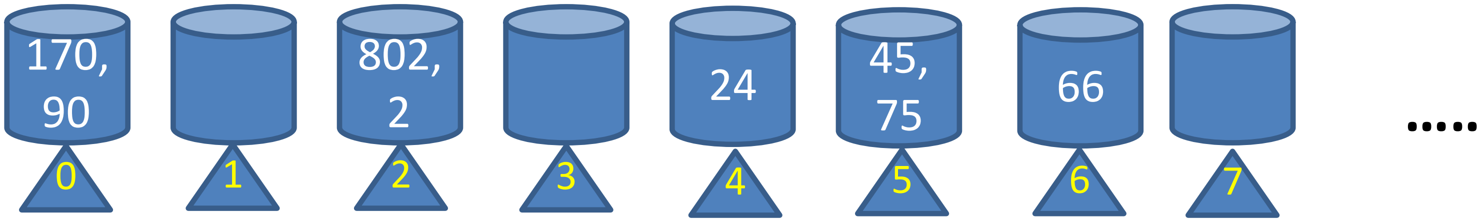
输入： *X*（元素下标从 1 至 *n* 的整数数组，每个元素有 *k* 位）

输出： *X*（排序后的数组）

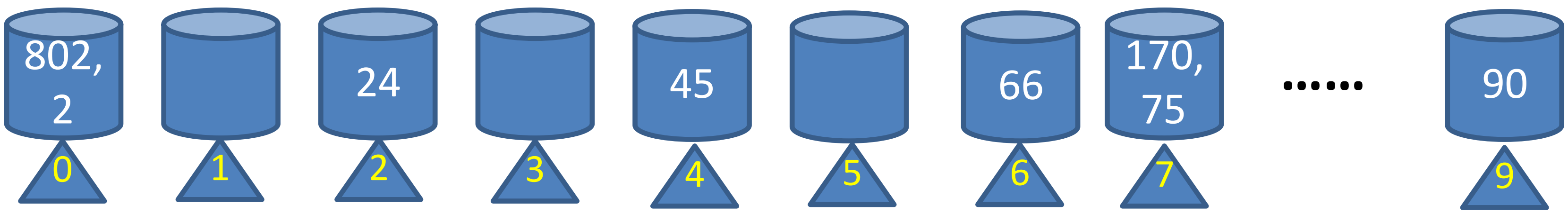
```
begin
    We assume that all elements are initially in a global queue GQ;
    {为简单起见，这里使用 GQ；当然也可以通过 X 本身来实现}
    for i := 1 to d do
    {d 是可能的数制，比如对于十进制，d = 10}
        Initialize queue Q[I] to be empty;
    for i := k downto 1 do                     降序
        while GQ is not empty do
            pop x from GQ ;
            d := the ith digit of x ;          分入桶
            insert x into Q[d] ;
        for t := 1 to d do
            insert Q[t] into GQ ;              收集
    for i := 1 to n do
        pop X[i] from GQ
end
```

图 6.6　算法 *Straight_Radix*

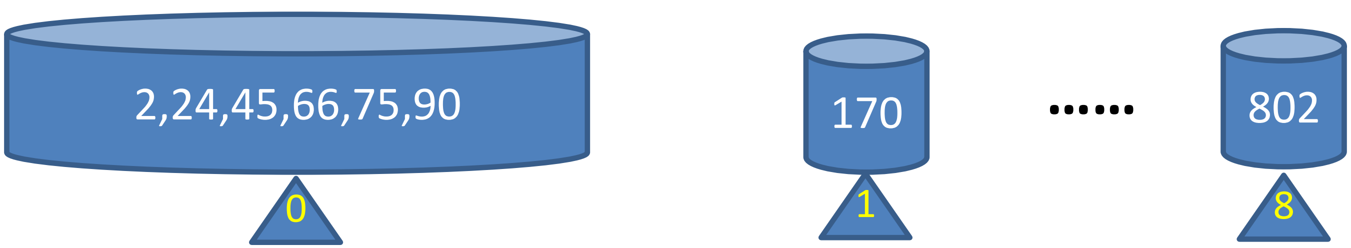
170, 45, 75, 90, 802, 2, 24, 66



170, 90, 802, 2, 24, 45, 75, 66

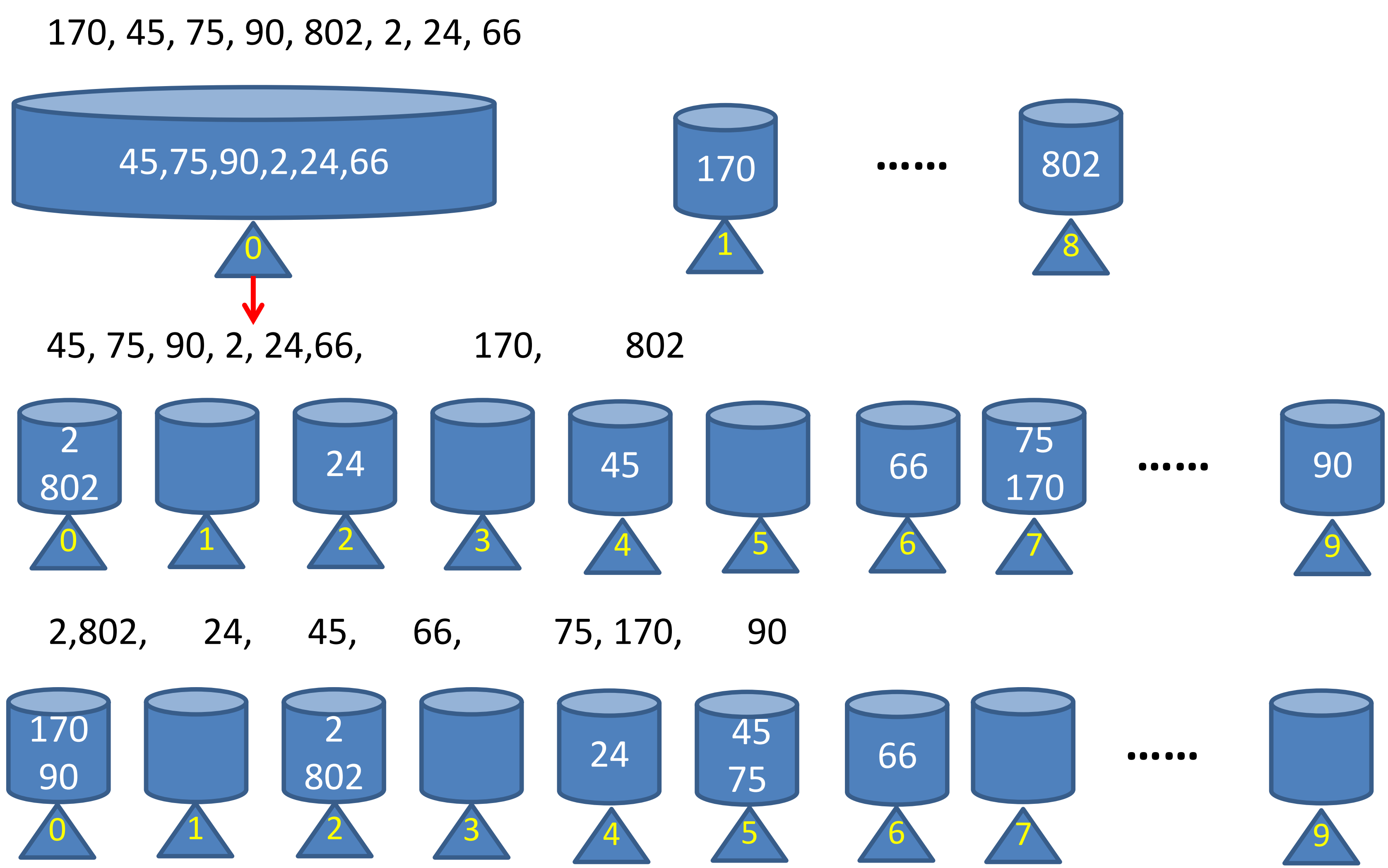


802, 2, 24, 45, 66, 170, 75, 90

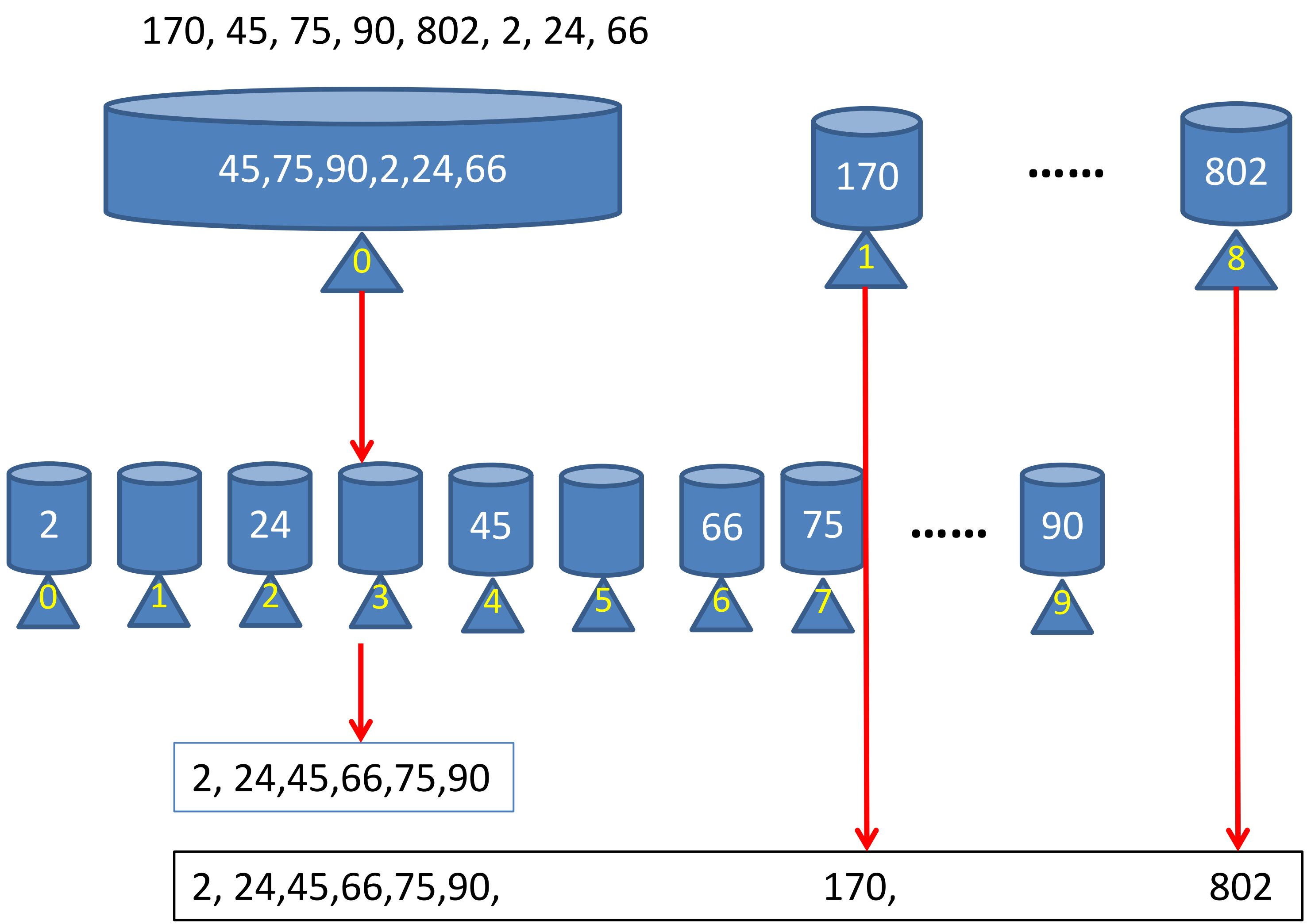


2, 24, 45, 66, 75, 90, 170, 802

对最高位相同的任意两个元素，在最后一步之前，他们已经按正确顺序排列了。



从高到低，如果直接按上一页slides的方法做，结果肯定是错的！必须要按递归做，见下一页,.



从高到低，必须按递归式，结果才正确。

Analysis

- 重收集桶的循环执行K次，K是最大数的位数(e.g.10进制下)
- 需要全局队列GQ 和 分桶队列Q[0]...Q[K-1].
- O((n+n)k) = O(nk);

缺点：依赖于数据的结构性.

插入排序
insertion sort

- example

6 5 3 1 8 7 2 4

Pseudocode

```
for i ← 1 to length(A)
    j ← i
    while j > 0 and A[j-1] > A[j]
        swap A[j] and A[j-1]
        j ← j - 1
```

插入排序的一个改进
Shell 排序

- example

[Donald Shell 1959]

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
input data:	62	83	18	53	07	17	95	86	47	69	25	28
after 5-sorting:	17	28	18	47	07	25	83	86	53	69	62	95
after 3-sorting:	17	07	18	47	28	25	69	62	53	83	86	95
after 1-sorting:	07	17	18	25	28	47	53	62	69	83	86	95

The running time of Shellsort is heavily dependent on the gap sequence it uses.
Best interval? open questions!

Shellsort is unstable(*changing relative order of equal elements*)
It is an adaptive sorting algorithm in that it executes faster when the input is partially sorted

the subarrays that Shellsort operates on are initially short; later they are longer but almost ordered:
In BOTH cases insertion sort works efficiently.

选择排序

selection sort

- example

64 25 12 22 11

11 25 12 22 64

11 12 25 22 64

11 12 22 25 64

11 12 22 25 64

基本思想:
每次都从右边剩下的n-k个数中选最小(大)的插入到第k个位置处.

输入list被分为两部分，前一部分总是排好序的sublist.

将64换成24,则最后一步也发生交换

```
/* a[0] to a[n-1] is the array to sort */
int i,j;
int iMin;

/* advance the position through the entire array */
/*   (could do j < n-1 because single element is also min element) */
for (j = 0; j < n-1; j++) {
    /* find the min element in the unsorted a[j .. n-1] */

    /* assume the min is the first element */
    iMin = j;
    /* test against elements after j to find the smallest */
    for ( i = j+1; i < n; i++) {
        /* if this element is less, then it is the new minimum */
        if (a[i] < a[iMin]) {
            /* found new minimum; remember its index */
            iMin = i;
        }
    }

    /* iMin is the index of the minimum element. Swap it with the current position */
    if ( iMin != j ) {
        swap(a[j], a[iMin]);
    }
}
```

Another example

	8
	5
	2
	6
	9
	3
	1
	4
	0
	7

选择排序

- 如果输入数据结构是(linked)list,性能更好.

many swapping

64 25 12 22 11

11 25 12 22 64

11 12 25 22 64

11 12 22 25 64

11 12 22 25 64

No swapping

64 25 12 22 11

11 64 25 12 22

11 12 64 25 22

11 12 22 64 25

11 12 22 25 64

performance

- Finally, selection sort is greatly outperformed on larger arrays by Θ(n log n) divide-and-conquer algorithms such as mergesort.
- However, insertion sort or selection sort are both typically faster for small arrays (i.e. fewer than 10–20 elements).
- A useful optimization in practice for the recursive algorithms is to switch to insertion sort or selection sort for "small enough" sublists.

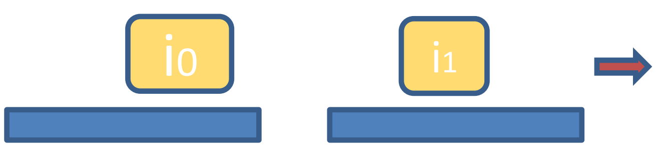
归并排序 merge sort

Top-Bottom

6 5 3 1 8 7 2 4

这是自顶向下递归，因此会分裂到长度为1的sublist再开始merge!

Major drawback:
Need a B for the merged list!



主函数

Merge into B

```
TopDownMergeSort(A[], B[], n)
{
    TopDownSplitMerge(A, 0, n, B);
}
CopyArray(B[], iBegin, iEnd, A[])
{
    for(k = iBegin; k < iEnd; k++)
        A[k] = B[k];
}
// iBegin is inclusive; iEnd is exclusive (A[iEnd] is not in the set)
TopDownSplitMerge(A[], iBegin, iEnd, B[])
{
    if(iEnd - iBegin < 2)                       // if run size == 1
        return;                                 //   consider it sorted
    // recursively split runs into two halves until run size == 1,
    // then merge them and return back up the call chain
    iMiddle = (iEnd + iBegin) / 2;              // iMiddle = mid point
    TopDownSplitMerge(A, iBegin,  iMiddle, B);  // split / merge left  half
    TopDownSplitMerge(A, iMiddle,    iEnd, B);  // split / merge right half
    TopDownMerge(A, iBegin, iMiddle, iEnd, B);  // merge the two half runs
    CopyArray(B, iBegin, iEnd, A);              // copy the merged runs back to A
}

//  left half is A[iBegin :iMiddle-1]
// right half is A[iMiddle:iEnd-1   ]
TopDownMerge(A[], iBegin, iMiddle, iEnd, B[])
{
    i0 = iBegin, i1 = iMiddle;

    // While there are elements in the left or right runs
    for (j = iBegin; j < iEnd; j++) {
        // If left run head exists and is <= existing right run head.
        if (i0 < iMiddle && (i1 >= iEnd || A[i0] <= A[i1]))
            B[j] = A[i0];
            i0 = i0 + 1;
        else
            B[j] = A[i1];
            i1 = i1 + 1;    }

}
```

Merge sort

- Bottom-Top

bottom up merge sort algorithm :
1 treats the list as an array of *n* sublists (called *runs* in this example) of size 1,
2 .iteratively merges sub-lists back and forth between two buffers.

主函数

```
/* array A[] has the items to sort; array B[] is a work array */
BottomUpSort(int n, int A[], int B[])
{
  int width;

  /* Each 1-element run in A is already "sorted". */

  /* Make successively longer sorted runs of length 2, 4, 8, 16... until whole array is sorted. */
  for (width = 1; width < n; width = 2 * width)
    {
      int i;

      /* Array A is full of runs of length width. */
      for (i = 0; i < n; i = i + 2 * width)
        {
          /* Merge two runs: A[i:i+width-1] and A[i+width:i+2*width-1] to B[] */
          /* or copy A[i:n-1] to B[] ( if(i+width >= n) ) */
          BottomUpMerge(A, i, min(i+width, n), min(i+2*width, n), B);
        }

      /* Now work array B is full of runs of length 2*width. */
      /* Copy array B to array A for next iteration. */
      /* A more efficient implementation would swap the roles of A and B */
      CopyArray(A, B, n);
      /* Now array A is full of runs of length 2*width. */
    }
}
BottomUpMerge(int A[], int iLeft, int iRight, int iEnd, int B[])
{
  int i0 = iLeft;
  int i1 = iRight;
  int j;

  /* While there are elements in the left or right lists */
  for (j = iLeft; j < iEnd; j++)
    {
      /* If left list head exists and is <= existing right list head */
      if (i0 < iRight && (i1 >= iEnd || A[i0] <= A[i1]))
        {
          B[j] = A[i0];
          i0 = i0 + 1;
        }
      else
        {
          B[j] = A[i1];
          i1 = i1 + 1;
        }
    }
}
```

Merge Sort的一个改进

- Natural merge sort

```
Start       : 3--4--2--1--7--5--8--9--0--6
Select runs : 3--4  2  1--7  5--8--9  0--6
Merge       : 2--3--4  1--5--7--8--9  0--6
Merge       : 1--2--3--4--5--7--8--9  0--6
Merge       : 0--1--2--3--4--5--6--7--8--9
```

any naturally occurring runs (sorted sequences) in the input are exploited!

Adavantage: 不需要像标准merge sort那样非得pass lbn 次.

MergeSort Performance

Class	Sorting algorithm
Data structure	Array
Worst case performance	O(n log n)
Best case performance	O(n log n) typical, O(n) natural variant
Average case performance	O(n log n)
Worst case space complexity	O(n) auxiliary